S15 M1 IAL

1. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a particle P.

$\mathbf{F}_1 = (2\mathbf{i} + 3a\mathbf{j})$ N; $\quad \mathbf{F}_2 = (2a\mathbf{i} + b\mathbf{j})$ N; $\quad \mathbf{F}_3 = (b\mathbf{i} + 4\mathbf{j})$ N.

The particle P is in equilibrium under the action of these forces.

Find the value of a and the value of b.

(6)

$RF = F_1 + F_2 + F_3 = 0$

$$\begin{pmatrix}2\\3a\end{pmatrix} + \begin{pmatrix}2a\\b\end{pmatrix} + \begin{pmatrix}b\\4\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$\therefore \begin{aligned} 2a + b &= -2 \\ 3a + b &= -4 \end{aligned}$$

$a = -2 \quad b = 2$

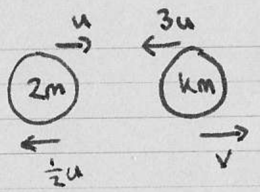
2. Particle A of mass $2m$ and particle B of mass km, where k is a positive constant, are moving towards each other in opposite directions along the same straight line on a smooth horizontal plane. The particles collide directly. Immediately before the collision the speed of A is u and the speed of B is $3u$. The direction of motion of each particle is reversed by the collision. Immediately after the collision the speed of A is $\frac{1}{2}u$.

(a) Show that $k < 1$

(6)

(b) Find, in terms of m and u, the magnitude of the impulse exerted on B by A in the collision.

(3)



Total Mom before = $2mu - 3kmu$

Total Mom after = $-mu + kmv$

CLM $\Rightarrow 2mu - 3kmu = -mu + kmv$

$\Rightarrow 3mu = kmv + 3kmu$

$\Rightarrow 3\cancel{m}u = k\cancel{m}(v + 3u)$

$\Rightarrow k = \dfrac{3u}{3u + v}$

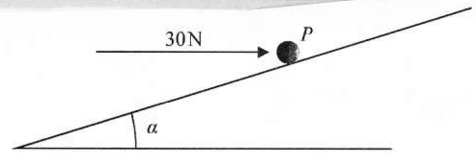
Since $v > 0$ $\quad 3u + v > 3u \quad \therefore k < 1$

b) Need to find Impulse exerted on A by B which is the same.

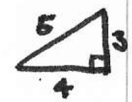
Mom A before $= 2mu$ $\quad \therefore$ Impulse $= 3mu$

Mom A after $= -mu$

3



$\tan\alpha = \frac{3}{4}$



$\sin\alpha = 0.6$

$\cos\alpha = 0.8$

A particle P of mass 2 kg is pushed by a constant horizontal force of magnitude 30 N up a line of greatest slope of a rough plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 1. The line of action of the force lies in the vertical plane containing P and the line of greatest slope of the plane. The particle P starts from rest. The coefficient of friction between P and the plane is μ. After 2 seconds, P has travelled a distance of 5.5 m up the plane.

(a) Find the acceleration of P up the plane.

(2)

(b) Find the value of μ.

(8)

$s = 5.5$

$u = 0$

v

a

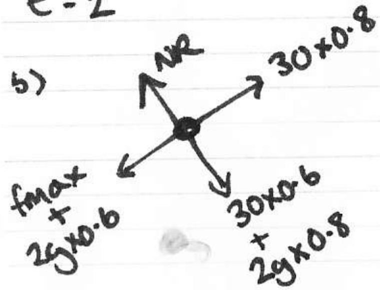
$t = 2$

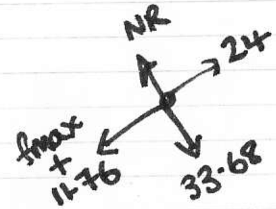
$s = ut + \frac{1}{2}at^2$

$5.5 = \frac{1}{2}a \times 4 \Rightarrow 2a = 5.5$

$\therefore a = 2.75$

b)





$R(\uparrow) = 0 \Rightarrow NR = 33.68 \Rightarrow f_{max} = 33.68\mu$

$R(\nearrow) = ma \Rightarrow 24 - 11.76 - 33.68\mu = 2 \times 2.75$

$\Rightarrow 33.68\mu = 6.74 \quad \therefore \mu = 0.2$

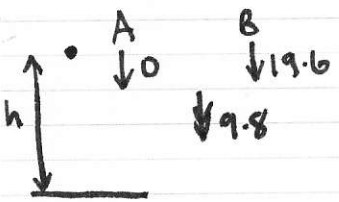
4. A small stone is released from rest from a point A which is at height h metres above horizontal ground. Exactly one second later another small stone is projected with speed 19.6 m s^{-1} vertically downwards from a point B, which is also at height h metres above the horizontal ground. The motion of each stone is modelled as that of a particle moving freely under gravity. The two stones hit the ground at the same time.

Find the value of h.

(7)

(A) $s = h$, $u = 0$, v, $a = 9.8$, $t = t$

(B) $s = h$, $u = 19.6$, v, $a = 9.8$, $t = t-1$



$s = ut + \frac{1}{2}at^2$

(A) $h = 4.9t^2$

(B) $h = 19.6(t-1) + 4.9(t-1)^2$

$\therefore \cancel{4.9t^2} = 19.6t - 19.6 + \cancel{4.9t^2} - 9.8t + 4.9$

$\therefore 9.8t = 14.7 \quad \therefore t = 1.5$ sec.

$h = 4.9t^2 = 4.9 \times \left(\frac{3}{2}\right)^2 = 11.025$

$h = 11.0$ m

2

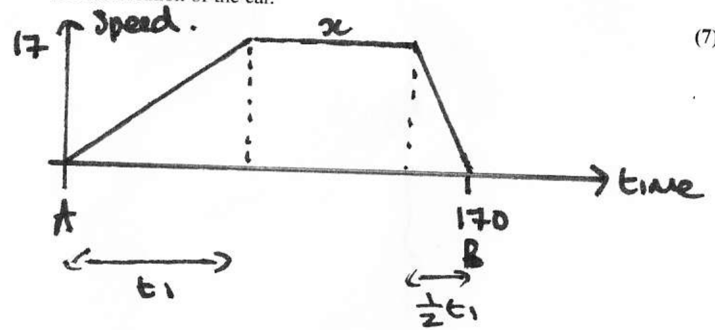
5. A car travelling along a straight horizontal road takes 170 s to travel between two sets of traffic lights at A and B which are 2125 m apart. The car starts from rest at A and moves with constant acceleration until it reaches a speed of 17 m s^{-1}. The car then maintains this speed before moving with constant deceleration, coming to rest at B. The magnitude of the deceleration is twice the magnitude of the acceleration.

(a) Sketch, in the space below, a speed-time graph for the motion of the car between A and B.

(3)

(b) Find the deceleration of the car.

(7)

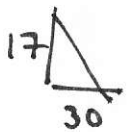


$\frac{(x + 170)}{2} \times 17 = 2125$

$x + 170 = 250 \quad \therefore x = 80$

$t_1 + \frac{1}{2}t_1 = 90 \Rightarrow 1\frac{1}{2}t_1 = 90 \quad \therefore t_1 = 60$

$\therefore t_2 = 30$



$\therefore$ dec = gradient (-ve)

$= \frac{17}{30}$

2

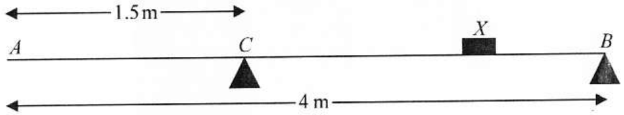


Figure 2

A plank AB has length 4 m and mass 6 kg. The plank rests in a horizontal position on two supports, one at B and one at C, where $AC = 1.5$ m. A load of mass 15 kg is placed on the plank at the point X, as shown in Figure 2, and the plank remains horizontal and in equilibrium. The plank is modelled as a uniform rod and the load is modelled as a particle. The magnitude of the reaction on the plank at C is twice the magnitude of the reaction on the plank at B.

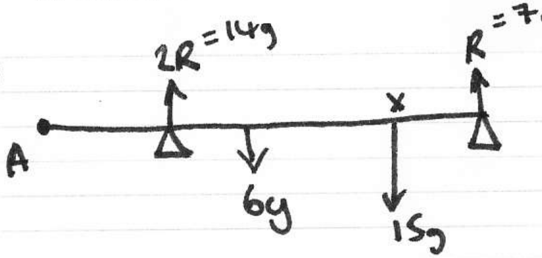
(a) Find the magnitude of the reaction on the plank at C.
(3)

(b) Find the distance AX.
(5)

The load is now moved along the plank to a point Y, between A and C. Given that the plank is on the point of tipping about C,

(c) find the distance AY.
(4)

a)



$R \uparrow = 0$

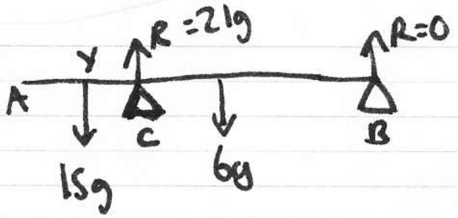
$3R = 21g$

$\therefore R = 7g$

A↻ $6g \times 2 + 15g \times AX = 14g \times 1.5 + 7g \times 4$

$\Rightarrow 15g \times AX = 37g \qquad AX = \frac{37}{15} = 2.4\dot{6}$

b)



$\uparrow = \downarrow \Rightarrow R = 21g$

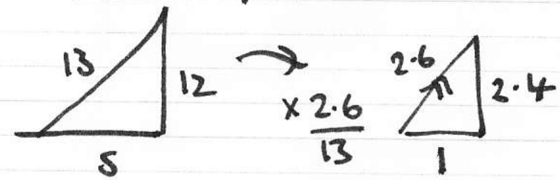
A↻ $15g \times AY + 6g \times 2 = 21g \times 1.5$

$15g \times AY = 19.5g \qquad AY = \frac{19.5}{15} = \underline{1.3\text{m}}$

7. A particle P moves from point A to point B with constant acceleration $(c\mathbf{i} + d\mathbf{j})$ m s^{-2}, where c and d are positive constants. The velocity of P at A is $(-3\mathbf{i} - 3\mathbf{j})$ m s^{-1} and the velocity of P at B is $(2\mathbf{i} + 9\mathbf{j})$ m s^{-1}. The magnitude of the acceleration of P is 2.6 m s^{-2}.

Find the value of c and the value of d.
(5)

$V_A\begin{pmatrix}-3\\-3\end{pmatrix} \quad V_B\begin{pmatrix}2\\9\end{pmatrix} \quad \text{change}\begin{pmatrix}5\\12\end{pmatrix}$



$\text{acc} = \begin{pmatrix}1\\2.4\end{pmatrix}$

8.

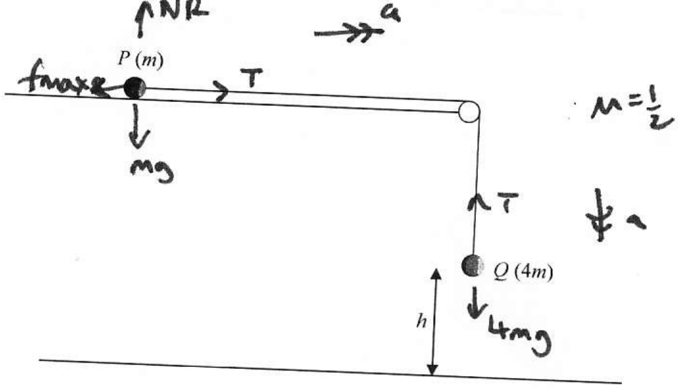


Figure 3

Two particles P and Q have masses m and $4m$ respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a rough horizontal table. The string lies along the table and passes over a small smooth light pulley which is fixed at the edge of the table. Particle Q hangs at rest vertically below the pulley, at a height h above a horizontal plane, as shown in Figure 3. The coefficient of friction between P and the table is 0.5. Particle P is released from rest with the string taut and slides along the table.

(a) Find, in terms of mg, the tension in the string while both particles are moving. **(8)**

The particle P does not reach the pulley before Q hits the plane.

(b) Show that the speed of Q immediately before it hits the plane is $\sqrt{1.4gh}$ **(2)**

When Q hits the plane, Q does not rebound and P continues to slide along the table. Given that P comes to rest before it reaches the pulley,

(c) show that the total length of the string must be greater than $2.4h$ **(6)**

$f_{max} = \mu NR = \frac{1}{2}mg$. $\quad \uparrow = \downarrow \; NR = mg$

(P) $T - \frac{1}{2}mg = ma$

$4mg - T = 4ma \quad +$

$\frac{7}{2}\cancel{m}g = 5\cancel{m}a \qquad a = \frac{7}{10}g = 6.86$

$T = ma + \frac{1}{2}mg = \frac{7}{10}mg + \frac{1}{2}mg = \frac{12}{10}mg$

$\therefore T = \frac{6}{5}mg$.

b) $s = h$ $\qquad v^2 = u^2 + 2as$

$u = 0$

$v \qquad v^2 = \frac{7}{5}gh$

$a = 6.86 \left(\frac{7}{10}g\right)$

$t =$ $\qquad \therefore v = \sqrt{1.4gh}$

c) after Q hits

$\frac{1}{2}mg \leftarrow \bigcirc \rightarrow 0$

$\overrightarrow{Rf} = ma \Rightarrow -\frac{1}{2}\cancel{m}g = \cancel{m}a$

$\therefore a = -4.9$

s

$u = \sqrt{1.4gh}$

$v = 0$

$a = -4.9 \qquad v^2 = u^2 + 2as$

$t \qquad 0 = 1.4gh - 9.8s$

$9.8s = 13.72h \quad \therefore S = 1.4h$.

total distance travelled by P is

$h + 1.4h = 2.4h \quad \therefore$ total length $> 2.4h$

otherwise Q could not hang below pulley